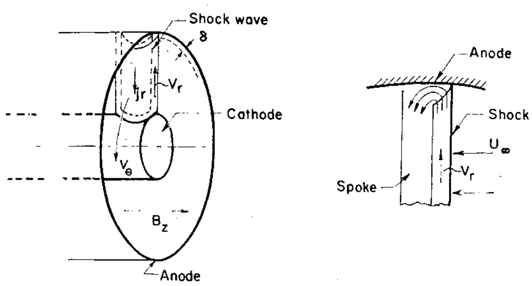

Fig. 2 Rotating arc spoke.

The corresponding azimuthal velocity is equal to

$$v_\theta = \left[\frac{r}{\rho} j_{e\theta} B_z\right]^{1/2} = \left[\frac{4 \times 10^{-2}}{3.4 \times 10^{-4}} 1.66 \times 10^6 \times 0.1\right]^{1/2} = 4.5 \times 10^3 \text{ m/sec}$$

Such a large velocity in a thin layer would result in a large friction force equal to

$$\mu \frac{\partial^2 v_\theta}{\partial r^2} \cong \mu \frac{v_\theta}{\delta^2} = 3 \times 10^{-4} \frac{4.5 \times 10^3}{(0.001)^2} = 1.35 \times 10^6 \text{ N/m}^3$$

which is order of magnitude larger than the force $j_r B_z$ driving the spoke. On this basis we should reject the model of a thin boundary layer with a spinning dense plasma.

Another possibility is that the plasma reflects from the wall and enters the spoke and moves inward (Fig. 2). This could be shown from the following consideration. The stagnation plasma pressure within the shock layer at the anode is equal to (density in the shock layer is equal to $\rho_s = (\gamma + 1)/(\gamma - 1)\rho = 4 \times 0.17 \times 10^{-4} = 0.68 \times 10^{-4}$ kg/m^3) the value calculated from the Newtonian formula

$$p_0 = \rho_s V_r^2 = 0.68 \times 10^{-4} \times 10^8 = 6.8 \times 10^3 \text{ N/m}^2$$

The pressure within the spoke is of order of

$$p_s = \rho_\infty U^2 = 0.17 \times 10^{-4} \times 10^8 = 1.7 \times 10^3$$

Because of the smaller pressure within the spoke, the pressure gradient could be larger than centrifugal force $\rho v_\theta^2/r$ and the plasma really should reflect from the wall and enter the spoke, leaving the spoke through the rear (Fig. 2).

There is experimental evidence that such a recirculation occurs[6] and the ion probe indicates that the gas is moving at the front of the spoke outward, followed by an inward motion of the plasma.

Spinning plasma entering the magnetic nozzle is further accelerated, like in the case of symmetric discharge. Energy losses should be equal to

$$m[V_r/2]^2$$

If final axial plasma velocity is equal to 10^4 m/sec, neglecting other losses, efficiency of the device with a rotating spoke would be 50%. This is an upper limit of efficiencies in this type of device.

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Effects of In-plane and Rotary Inertia on the Frequencies of Eccentrically Stiffened Cylindrical Shells

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Accurate determination of natural frequencies of vibration of stiffened shells is essential for various engineering applications, e.g., estimation of fatigue life, supersonic flutter analysis of rockets and missiles, etc. In this Note the simple arbitrary mode defined by

$$u = \bar{u} \cos\frac{m\pi x}{a} \cos\frac{ny}{R} \sin\omega t$$
$$v = \bar{v} \sin\frac{m\pi x}{a} \sin\frac{ny}{R} \sin\omega t \quad (1)$$
$$w = \bar{w} \sin\frac{m\pi x}{a} \cos\frac{ny}{R} \sin\omega t$$

is used for the axial, circumferential, and radial displacements respectively in the strain and kinetic energy expressions of Refs. 1–3 to study the influence of various inertia terms on the invacuo frequencies of vibration of simply supported eccentrically stiffened circular cylindrical shells and to examine the efficacy of 1) stiffener discreteness as compared to stiffener smearing and 2) stiffener configuration.

For the discrete stiffener analyses it is assumed that the $2L$ stringers and the $(K + 1)$ rings are located at positions determined respectively by

$$y_l/R = (2l - 1)/2L, \; l = 1,2, \ldots 2L$$
$$x_k/a = k/K, \; k = 0,1,2, \ldots K \quad (2)$$

This type of stiffener distribution has the advantage that their axial and radial displacements are zero when the circumferential nodes are a multiple of the number of stringers, and their circumferential and radial displacements are zero when the axial nodes are a multiple of the number of rings. Simple support boundary conditions are satisfied by this choice of stiffener distribution. The details of these analyses for the "smeared" and "discrete" stiffener cases are given in Refs. 1–2, respectively and summarized in Ref. 3.

Table 1 Properties of shells for numerical examples

	Ref. 4	Ref. 1	Ref. 2	
a, in.	40	23.75	24.00	38.85
r, in.	20	9.55	9.537	7.657
t, in.	0.04	0.028	0.0256	0.01826
E, E_s, E_r, 10^6 psi	10	10.5	10	29
ρ lb/in.^{3}	0.0998	0.095	0.0975	0.2819
ν	0.3	0.3	0.315	0.3
b_s, b_r, in.	...	0.096	0.1118	0.0409
h_s, h_r, in.	...	0.302	0.2262	0.3981
L	...	60	60	4
K	...	25	...	...
d, in.	...	1	...	...
l, in.	...	1	...	...

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Table 2 Calculated natural frequencies for an unstiffened shell with and without various inertia terms[a]

	Present analysis			Ref. 4	Ref. 4
n^a	(i)	(i)(iii)	(i)(ii)(iii)	(i)	(i)(ii)
2	3741.24	3741.23	3354.00	3741.17	3362
4	1314.71	1314.70	1271.00	1314.16	1270
6	669.12	669.11	659.30	666.97	657
8	536.76	536.74	532.40	532.22	527
10	652.35	652.31	649.00	646.66	643

[a] (i) radial inertia, (ii) inplane inertia, (iii) rotary inertia.

Calculations were performed on three shell geometries for which experimental and/or theoretical results were already available. Their properties are listed in Table 1.

Table 2 gives the comparison between the results obtained for the unstiffened shell analysed in Ref. 4, 1) by the present analysis with and without various inertia terms, 2) the analysis of Ref. 4 neglecting inplane inertia terms, and 3) the results of Ref. 4. It is seen that inplane inertia can have a significant effect as compared to rotary inertia.

Table 3 gives the comparison of minimum frequencies for an eccentrically stiffened shell of Ref. 1 with various "smeared" and "discrete" stiffening configurations. Eccentricity alters the frequencies considerably. Of all possible configurations considered, internal rings yielded much higher frequencies when compared with the others (see Fig. 1). Figure 2 illustrates the influence of in-plane and rotary inertias for various values of n for a shell stiffened internally with rings.

Stringers, when attached internally or symmetrically yielded frequencies which are seen to be lower than the corresponding unstiffened shell probably because the stringer contribution to the kinetic energy is greater than its contribution to the strain energy for low values of m. With larger values of m, however, increase of stiffness prevails over that of mass resulting in higher frequencies, as shown in Refs. 2 and 5. Also included in Table 3 are the minimum frequencies of the shell when the stiffeners are treated as discrete elements. The results are in very good agreement with those for smeared stiffeners.

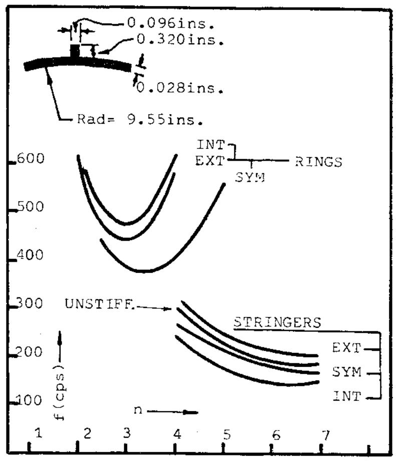

Fig. 1 Frequency spectrum, showing the effect of stiffener configuration and eccentricity.

Table 4 shows the comparison of frequencies of a shell stiffened by four internal stringers analysed in Refs. 2 and 5 with the present analysis. Comparable results are given for the same shell with 60 external stringers in Table 5. The results from the present analysis are somewhat lower than those of Ref. 2, probably due to the fact that in-plane inertias were neglected in the latter calculations. Surprisingly, there is very good agreement, regardless of whether the stringers are treated as smeared or discrete, even when they are few in number, and particularly for the minimum frequency. Thus at least for the cases considered with $m = 1$, the assumption of discrete stiffening seems to have little advantage over smeared stiffening whether the stiffeners are densely or sparsely spaced. In fact, because of the stiffener distributions assumed in Eq. (2), the frequencies by smeared and discrete analysis are identical for odd values of n, whereas there

Table 3 Minimum frequencies (cps) for a cylindrical shell with various stiffening configurations ($m = 1$)[a]

No.	Case	n	S(i)	S(i)(iii)	S(i)(ii)(iii)	D(i)(ii)(iii)
1	Unstiffened	6	185.35	185.34	182.68	182.68
2	Stringers—external	7	205.40	202.41	202.08	204.75
3	Stringers—symmetric	7	169.50	168.97	167.57	167.56
4	Stringers—internal	6	150.49	149.03	146.61	147.00
5	Rings—external	3	480.79	479.91	452.57	450.33
6	Rings—symmetric	3	417.12	416.95	393.71	390.99
7	Rings—internal	3	509.98	509.05	482.01	480.04
8	Stringers & rings—external	3	484.28	483.11	458.85	457.89
9	Stringers & rings—symmetric	4	375.60	375.22	363.68	366.32
10	Stringers & rings—internal	3	436.28	435.22	411.94	411.68
11	Stringers—internal, rings—external	3	435.75	434.70	408.20	407.84
12	Stringers—external, rings—internal	3	473.01	471.87	451.04	450.05

[a] S = Smeared, D = Discrete, (i) radial inertia, (ii) inplane inertia, (iii) rotary inertia.

Table 4 Frequencies of a shell stiffened with four internal stringers

n	Present		Expt. Ref. 5	Egle & Sewall (Ref. 2) discrete		
	Discrete	Smeared		Symmetric	Antisymmetric	Unstiffened
2	314.61	315.31	...	...	...	...
3	158.72	158.72	...	169	169	171
4	100.27	102.21	100	103	108	108
5	93.09	93.09	87	94.7	94.7	98.1
6	115.00	113.91	104	109	116	117
7	144.00	144.00	137	145	145	151
8	179.71	185.26	176	183	192	194
9	233.26	233.26	224	236	236	243
10	296.81	287.38	295	278	297	300

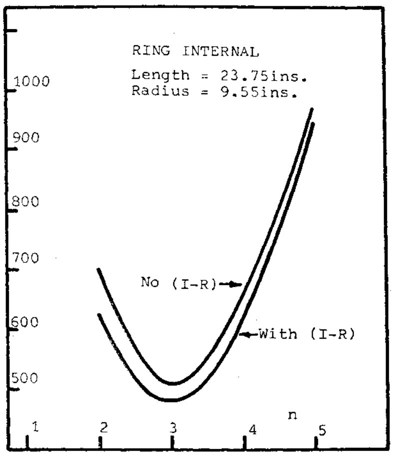

Fig. 2 Frequency spectrum showing the effect of in-plane and rotary inertia (I-R) for a shell stiffened internally with rings.

is only a small difference for even values of n. Clearly it would be of interest to extend these analyses to see whether the previous conclusions have more general validity. The frequency spectrum which is a smooth curve in the smeared case has a slight wavy pattern if the stringers are treated as discrete elements (see Fig. 3).

Thus it is shown that a one term solution with a proper choice of stiffener distribution and including the effects of in-plane and rotary inertias yields results for the natural frequencies which are in good agreement with existing experimental data and more complicated theoretical analyses using multiterm solutions (e.g., Ref. 2).

Table 5 Frequencies of a shell stiffened with 60 external stringers (m = 1)

n	Present discrete	(Ref. 2) discrete	(Ref. 2) smeared
2	666.98	736.5	736.3
3	424.89	445.3	445.1
4	297.10	304.0	303.9
5	229.56	231.8	231.8
6	197.13	197.7	197.9
7	187.83	188.2	188.6
8	194.77	196.0	196.7
9	213.33	216.1	217.0
10	240.41	245.2	246.3

The omission or inclusion of any particular effect mentioned can be studied by means of a single computer program.

The intention is to develop the program, using the information presented in this paper, for supersonic shell flutter analyses. To that end, it was decided at an early stage that the generalized aerodynamic forces would more easily be determined using the single, simple trigonometric mode chosen than from a more complex set of normal modes. Such a set could be obtained from a vibration analysis involving many degrees of freedom as in Ref. 2 but this has not been attempted here.

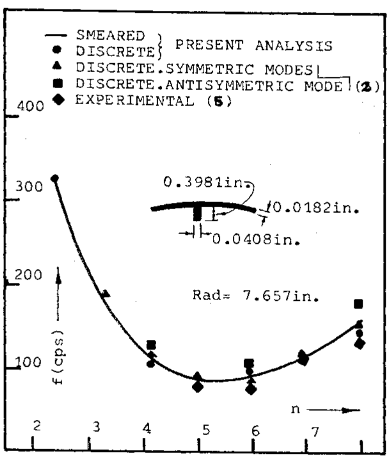

Fig. 3 Frequency spectrum, showing the effects of discrete and smeared stiffening for a shell stiffened internally with four stringers.

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Ammonium Perchlorate Combustion Analogue: Ammonia-Chlorine Dioxide Flames

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It is now generally accepted that ammonium perchlorate decomposes by proton transfer[1] to yield ammonia and perchloric acid

$$NH_4ClO_4 \rightleftharpoons NH_3 + HClO_4 \quad (1)$$

At pressures above a few atmospheres, it is considered[2,3] that the combustion of solid propellents based on this oxidizer involves two flame zones. The first is a premixed flame supplied by the thermal decomposition of the ammonium perchlorate. This premixed ammonia-perchloric acid flame is markedly oxidizer rich,

$$NH_3 + HClO_4 \rightarrow \tfrac{1}{2}N_2 + \tfrac{3}{2}H_2O + HCl + \tfrac{5}{4}O_2 \quad (2)$$

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